KERALA MATHEMATICS

History and Its Possible Transmission to Europe

Edited by

George Gheverghese Joseph
University of Manchester, U.K.

Published with the assistance of
Kerala Council for Historical Research
Thiruvananthapuram - 695 003
and
Arts and Humanities Research Board, U.K.

B.R. PUBLISHING CORPORATION
Delhi - 110 052

CONTENTS

9

EARLY TRANSMISSIONS OF INDIAN MATHEMATICS

Sreeramula Rajeswara Sarma
University of Aligarh, India

1.0 Introduction

Scientific activity takes place in all cultures, sometimes independently of one another, other times from inspiration received from other cultures. The Renaissance notion of Greece being the sole origin of all sciences began to be shaken already towards the end of the nineteenth century, thanks to the efforts of early Orientalists. In recent times, the very lucid and therefore widely accessible *The Crest of the Peacock* contributed greatly in emphasizing the non-European roots of modern mathematics. It is understandable that as a reaction to the prevailing notion of Graeco-centrism, or Euro-centrism, other cultures too make similar claims; thus we have now Sino-centrism, Islam-centrism and Indo-centrism. One must naturally be cautious about these new movements; especially the last mentioned variety in its endeavour to read every scientific concept into the *Ṛgveda.*

But then how do we establish the origin of a scientific notion? One way is by the first occurrence of the notion in a particular culture, assuming of course that the chronology is secure. It may at least show that culture A, at an early period, was producing valuable scientific ideas. But priority is one thing; being the sole origin is another thing. The same idea may have occurred to other people, say to culture B, quite independently, even if chronologically later. Therefore, to show that A transmitted the idea to B, or that B borrowed the idea from A, one must be able to trace the path of transmission from A to B and identify the facilitators of this transmission.

Sometimes, the receiving culture acknowledges the debt. For example, the Arabs have handsomely acknowledged that the decimal place value system and the associated symbols and operations came from India.[1] In his recent work *Wissenschaft und Technik im Islam,* Fuat Sezgin emphasizes that the Arab scholars of the medieval period scrupulously mention their sources.[2]

Indians were not far behind in their expressions of scholarly gratitude. Early Sanskrit texts on astronomy refer to the *Yavanas* (meaning Greeks) as masters in that subject. In an oft-quoted verse, Varāhamihira states that the Greeks, being great experts in astral science, are honoured like Ṛiṣis.[3] Similarly, titles like *Romakasiddhānta, Yavanajātaka* suggest that these texts were derived from outside. In 1370 when the Jaina monk Mahendra Sūri wrote the very first Sanskrit manual on the astrolabe, he clearly states that this science of the astrolabe was derived from *Yavanas* (this time meaning Muslims).[4] A whole class of Sanskrit astrological works produced in this and the subsequent centuries on the basis of Islamic astrology were clearly designated as *Tājika.*[5]

Sometimes philology can come to the aid. For example, when we trace the derivation of the word *zero*, through medieval Latin *zephirum*, Arabic *ṣifr* to Sanskrit *śūnya,* or of the term "sine" through Latin *sinus*, through Arabic *jayb* to Sanskrit *jīvā*, this reflects the transmission of these terms and the concepts associated with them from India to Europe via the Middle East.[6] Loan words from foreign languages can indicate transmission of ideas; for example, the Greek words *hora, kendra* etc. in some Sanskrit texts.[7]

Sometimes the alien element clearly stands out. For example, oldest sources like the *Vedāṅga-jyotiṣa, Sūriyapaṇṇatti, Mahābhāṣya, Arthaśāstra, Śārdūlakarṇāvadāna* consistently state that the longest day has the duration of 18 *muhūrtas* (= 14;24 hours) as if this maximum value is true for all

geographical latitudes. Albrecht Weber suggested that this value, together with a host of other elements, was borrowed from Babylonia since Ptolemy gives almost the same duration (14 h 25 m) for the longest day at Babylon.[8]

But oftentimes, we can only talk of the "possibility" of transmission and not show conclusively that a transmission had actually occurred.[9] It is not my intention to give an overview of all the transmissions that took place before the advent of Kerala mathematics. These have been well documented, most notably by the author of *The Crest of the Peacock.* What I wish to discuss now are the problems one encounters when one tries to map the process and the paths of transmission. I shall do this through a few examples.

2.1 Development and Transmission of the Decimal System

I may begin with the most important of such transmissions, namely the transmission of the decimal place value system with the nine digits and zero, which is universally employed today. It is also universally acknowledged that it came from India. But problems start cropping up when one wishes to examine the different stages in the process of its development in India and in its of transmission from India.

There are four distinct elements that constitute the decimal place value system. Only when we are sure of the chronology of these elements within India, we can discuss their transmission outside. These elements are (1) counting with the base of ten, (2) the concept of zero, (3) the notion of place value, and (4) the symbols for the nine digits and zero.

Let us take the counting with the base of ten first. In the *Vājasaneyī Saṃhitā* of the Black *Yajurveda* and allied texts, there occur frequently series of decuple terms starting from *eka, daśa, śata* reaching up to *parārdha* which designates the thirteenth—and at that time, the ultimate— decimal place. Jaina canonical writings and Buddhist narratives contain much larger series. This terminology is quite fundamental to India; such large series did not exist anywhere outside India.[10]

Next, about the concept of zero (*śūnya*), Bibhutibhushan Datta and Avadhesh Narayan Singh have shown, as early as

1935, in their *History of Hindu Mathematics* that *śūnya* occurs for the first time in Piṅgala's *Chandaḥsūtra.*[11] Adequate attention has not been paid to this evidence in the prevailing historiography. I may, therefore, dwell on this aspect a little.

While teaching how to calculate the number permutations of a verse foot containing a certain number of syllables—each syllable being either short or long—, Piṅgala lays down a procedure in which certain steps in the calculation are to be marked with *dviḥ* (two) and certain others with *śūnya* (zero). That is to say, Piṅgala uses the symbols for zero and two as markers for distinguishing between two kinds of operations. The symbol of two marks the place where there is an even number which can divided by 2 and where squaring has to be done later; the symbol for zero marks the stages where there is an odd number and consequently absence of halving, and where a multiplication by 2 has to be performed. The whole computation can of course be done without any markers at all or with any other symbols. However, the fact that Piṅgala uses these two markers, that too in a meaningful way, shows that at Piṅgala's time there existed a well recognised symbol for *śūnya*, but we do not know what that symbol was. A symbol presupposes a concept. What kind of mathematical concept lay behind this symbol for *śūnya*? From Piṅgala's use, it may appear that *śūnya* meant here the absence of an operation, akin to the grammarian's *lopa*. But is that all, or does *śūnya* here imply place value as well?

In this connection, it is useful to consider the view of Joseph Needham who states that "Place value could and did exist without any symbol for zero. But zero symbol as part of the numerical system never existed and could not have come into being without place value."[12] Therefore, Piṅgala's employment of zero symbol pre-supposes place value. That it can only be decimal place value needs no emphasis. Therefore, the invention of the decimal place value system along with the concept and symbol for zero must antedate considerably Piṅgala's mention of the zero symbol. But when did Piṅgala live?

Albrecht Weber in 1863 and, following him, some scholars recently,[13] argued that the eighth chapter of the *Chandaḥsūtra* where *śūnya* occurs is not genuine because certain parts of this chapter do not occur in some manuscripts to which Weber had access. I have countered this by showing that "the eighth chapter containing combinatorics and the first mention of *śūnya* is not a late interpolation but an integral part of Piṅgala's *Chandaḥsūtra.*"[14] It is not necessary to repeat my arguments here.

But Piṅgala's time is difficult to determine in absolute terms, and here lies one of the serious problems of Indian historiography. All that can be said is that there exists a close similarity in the method of exposition and in the mode of notation between the *Chandaḥsūtra* on the one hand and the *Aṣṭādhyāyī* and the *Vedāṅga-jyotiṣa* on the other. This affinity in style places the *Chandaḥsūtra* about 400 BC; in any case certainly before the commencement of the Christian era. Therefore it will not be rash to conclude that the decimal place value system with a symbol for zero developed in India before the beginning of the Christian era.

Babylonians had place value in their sexagesimal system, though not the zero. The Chinese too had a "fundamentally decimal place value" much before the third century AD, but they too did not have the zero. Nevertheless, could the Babylonian notion of place value and the Chinese decimal system have played any part in the development of the decimal place value in India? It is not impossible, though firm evidence is lacking. This much, however, seems to be certain: the full development of the decimal place value in its modern form with the use of zero took place in India before the beginning of the Christian era— not in the fifth century as most of the current literature states— and it is from India that the system was transmitted to other countries.

Bibhutibhusan Datta and Avadhesh Narayan Singh in their above-mentioned book, and several subsequent writers enumerated the literary evidences for decimal place value in the early centuries of the Christian era, such as *Anuyogadvāra* (probably 100 BC), Sphujidhvaja's *Yavanajātaka* (269/70 AD), *Lokavibhāga* (458 AD) and so on.

But what about the written symbols for the nine digits and especially for zero? It is one of the ironies of history of Indian science that the earliest written records containing a symbol for zero are found not in India but outside. In a valuable paper, G. Coedès discussed the numerical expressions and symbols occurring in the inscriptions of South-East Asia between the fifth and eighth centuries AD.[15] In one of these inscriptions, viz. in a Sanskrit inscription found at Bayan in Cambodia, the Śaka year 526 (= 604 AD) is recorded in numerals as "526" and in word symbols as "*rasa-dasra-śaraiḥ śakendravarṣe*". The decimal place value is clearly evident both in the numerals and in the chronogram with the word symbols. Therefore, it is argued that this is the earliest occurrence of numerals with place value and that, therefore, the modern system of decimal place value originated in South-East Asia from where it spread to India.

But Datta and Singh drew attention to an earlier occurrence in India itself. This is the Mankhani charter found at Sankheda in Gujarat. Here the year is written in numerals in decimal notation as "346"; the year is in the *Kalachuri-Chedi* Era, corresponding to 594-6 AD.[16] Thus there is conclusive evidence for the use of numerical symbols with place value in India, even before the inscription in Cambodia.

Still doubts persist about the origin of the zero symbol and its use with decimal place value. In the paper mentioned above, Coedès refers to two inscriptions that contain zero symbols. A Khmer inscription at Sambor in Cambodia from AD 683 records the Śaka era 605, representing the zero with a thick dot (•); three years later, a Malay inscription at Palembang in Sumatra from AD 686 contains the numbers 60 and 608 Śaka, where zero is shown by a small circle as we write today (o).

Does this warrant the conclusion that the symbol for zero developed in South-East Asia and then spread to India, as Needham argues? It is well worth quoting Needham in full:

> "Coedès does not believe that the south-east Asian inscriptions indicate an east Asian origin for the symbolic word system..., but rather that the Hinduising settlers of south-east Asia already had symbolic words and the old numerals when they first went there, or at any rate were soon followed by them. So far so good, but we are free to consider the possibility (or even probability) that the written zero symbol, and the more reliable calculation which it permitted, really originated in the eastern zone of Hindu culture where it met the southern zone of the culture of the Chinese. What ideographic stimulus could it have received at that interface? Could it have adopted an encircled vacancy from the empty blanks left for zero on the Chinese counting boards ? The essential point is that the Chinese had possessed, long before the time of *Sun Tzu Suan Ching* (late +3rd Century) a fundamentally decimal place-value system. It may be, then, that the 'emptiness' of Taoist mysticism, no less than the 'void' of Indian philosophy, contributed to the invention of a symbol for *śūnya,* i.e. the zero. It would seem, indeed, that the finding of 'the first appearance of the zero in dated inscriptions on the border line of the Indian Chinese culture-areas can hardly be a coincidence."[17]

Unfortunately, there is more wishful thinking than reasoned argument in this passage. In his eagerness to prove the Chinese inspiration for the invention of zero, Needham completely ignores the occurrence of *śūnya* in Piṅgala's *Chandaḥsūtra.* He ignores too the other Sanskrit inscriptions discussed by Coedès in the same article. But, if the symbol for zero was invented in South-East Asia in the seventh century under inspiration from China—which did not until then had a symbol of her own— and then transmitted to India, why then were two symbols invented at the same time in South-East Asia, a small circle and a thick dot? It should also be noted that there are several other inscriptions in this region which were composed in Sanskrit; which show that the Indian settlers brought with them Sanskrit language, Indian calendar, Indian Śaka era, decimal place value, Sanskrit word numerals, Indian symbols for the digits 1 to 9. Surely they would also have brought the zero symbol with them.

The fact is that these two symbols developed in India, not at the same time, but one after the other. Of these, the dot (*bindu*) is the earlier one. Later it was enlarged into a circle (*chidra* or *randhra*) for purely practical reasons; so that it could be clearly recognised as a symbol and not mistaken for an accidental dot. An analogous shift occurred with *anusvāra* in Kannada-Telugu script. In early inscriptions, the *anusvāra* was represented by a small dot, often hardly visible on the copper plates. Gradually it was replaced by a small circle, which is called in Telugu *sunna* (from Sanskrit *śūnya*) since it was identical with the symbol for *śūnya*.

Since these two symbols appear in inscriptions of South-East Asia in the seventh century, it is reasonable to conclude that both the symbols were in use in India at this time and that the circular symbol must have developed by this century at the latest. The change, however, was not simultaneous everywhere. The dot seems to have prevailed longer in Gandhāra and Kashmir regions. Thus in the mathematical text known as the *Bakhshālī Manuscript*[18] and in the unique copy of an anonymous commentary on Śrīdhara's *Pāṭīgaṇita,*[19] both written in an early form of Śāradā script some time after the ninth century, the zero is represented by a dot (.). Moreover, even after the dot was replaced by the circle, the symbol continued to be called *bindu* or *śūnya-bindu.*[20]

While the symbol for zero was gradually transforming itself from a dot to a circle, the symbols for the nine digits too underwent changes. These changes can be seen in epigraphic records in India as well as in South-East Asia.

2.2 Transmission to China

With the spread of Buddhism in China, there took place a massive exchange of scholars between India and China. Buddhist pilgrims visited India and carried back large quantities of manuscripts with them; Indian scholars went to China where they were active in translating Buddhist texts into Chinese. These Chinese and Indian scholars were primarily interested in religious texts, especially of Buddhism, and the reports generally talk only about religious and philosophical texts. But they must have also carried with them some notions or texts about Indian systems of numeration, mathematics and astronomy. About this activity there are some stray references.

The catalogue of the Sui dynasty, completed in 610 AD, mentions some Chinese translations of Indian works on astronomy, mathematics, and medicine. These works are now lost, but their very existence shows that towards the end of the sixth century, the Chinese had gained some knowledge of Indian astronomy, mathematics and pharmaceutics.

Records of the Tang dynasty indicate that from 600 AD onwards Indian astronomers were employed at the Astronomical Board of Chang-Nan to teach the principles of Indian astronomy and calendar. One of the Indians named Gautama Siddhārtha was reported to have constructed a calendar, based on the Indian *Siddhāntas*. This calendar contains sections on Indian numerals and arithmetical operations and sine tables at intervals of 3°54' for a radius of 3438 units, which are precisely the values given in the Indian astronomical texts. There survives a block print text, which contains Indian numerals, including the use of a dot to indicate zero.[21]

Strangely enough, neither the dot for zero found in this calendar prepared by Gautama Siddhārtha, nor the dot and circle for zero found in inscriptions in what Needham refers to as the "southern zone of the culture of the Chinese" in the seventh century seem to have had any impact in China proper, because the Chinese did not start using the zero symbol until the mid-thirteenth century when it appears for the first time in the work of Qin Jiushao. It is rather intriguing why the Chinese took such a long time (nearly six centuries) to use the zero symbol which was brought up to their doorstep, so to speak. Here is a case of clear transmission having no impact in the receiving culture. The same appears to be the case with the other elements of Indian mathematics and astronomy introduced into China in the Tang period.[22]

To conclude, there are extensive records about the transmission of Buddhist texts and ideas from India to China. There are also records of Sanskrit astronomical and mathematical texts being translated into Chinese. In the reverse direction, we have no records of transmission from China to India. Still transmission of ideas must have occurred as did the transmission of objects. A number of parallel developments are also known. But one looks in vain for firm evidence of interaction between these two cultures in the realm of mathematics. This then is a grey area in the history of intellectual exchanges between these two cultures.

2.3 Transmission to the West

Fortunately, such uncertainties do not occur in the case of transmission to the Middle East or the Arab culture area. During the reign of the second Abbāsid Caliph al-Mansūr (753-774), the Indian province of Sindh passed under the control of the Caliphate and embassies from Sindh started visiting Baghdad. Sometimes these were accompanied by scholars. In his *India,* Al-Bīrūnī states : "These star-cycles as known through *the canon* of Alfazârî and Ya'ḳûb Ibn Târiḳ, were derived from a Hindu who came to Baghdad as a member of the political mission which Sindh sent to the Khalif Almansûr, A.H. 154 (= AD 771)."[23] Others also report about this mission. Ibn al-Adamī states in the preface to his astronomical tables entitled *Naẓm al-iqd* that "an Indian astronomer, well versed in the sciences, visited the court of al-Mansūr, bringing with him tables of the equations of planets according to the mean motions, with observations relative to both solar and lunar eclipses and the ascension of signs. Abu-Māsher of Balkh, an astrologer at the court of al-Mansūr, mentions that he derived the knowledge of the Hindu great cycle of the '*kalpa*' from an Indian astronomer. The name of this Indian astronomer is written variously as 'Kankaraf', 'Kankah', or 'Cancah', 'Kenker'...."[24] David Pingree avers that the name was Kanaka and that "the later Arabic writers slowly developed an elaborate mythology concerning Kanaka's role in the history of astronomy," attributing to this mythical figure scholarship and skills of diverse kinds.[25]

Perhaps the real word was not "Kanaka" but "Gaṇaka"— not a proper name but a generic name for the astronomer. Probably this word referred not to one particular astronomer who visited Baghdad in 771, but collectively to all Hindu astronomers or learned people who visited Baghdad about this time. This would explain the diverse qualities attributed to this Gaṇaka who may have, in reality, represented different persons.[26]

Be that as it may, the Indian decimal place value system reached the Middle East not through this visit of Kanaka / Gaṇaka but at least a century earlier, for already in 662 AD the Nestorian Bishop Severus Sebokht sings the praise of Indian decimal numbers, in a passage that is oft-quoted.[27]

The embassies from Sindh to the court of Al-Mansūr resulted in the transmission of many more scientific ideas, besides the decimal system and arithmetical operations with this

system. At the court of Al-Mansūr, Brahmagupta's *Brāhmasphuṭasiddhānta* and *Khaṇḍakhādyaka* were rendered into Arabic respectively by Al-Fazārī and Ya'qūb ibn Tārikh. These are not literal translations but adaptations and came to be known under the names *Sindhind* and *Al-Arkand.* Through these works, Arab scholarship became acquainted for the first time with mathematical astronomy, a few decades before the discovery of Greek astronomy. In the next century, around 820 AD, Muḥammad ibn Mūsa al-Khwārizmī summarized the knowledge gained thus far in three works, one on arithmetic, another on algebra and the third on mathematical astronomy. In the first work on arithmetic entitled *Kitāb al-jam'wal tafrīq bi ḥisāb al-Hind,* al-Khwarizmī explained the arithmetical operations of addition, subtraction, multiplication, division and the extraction of square roots according to the Indian system. Within a century, this treatise was superseded in the Eastern Islamic world by other introductions to Indian arithmetic. However, the work was still available in the Moorish Spain in the twelfth century where it was translated into Latin, under the title *Liber algorismi de numero Indorum,* "The Book of al-Khwārizmī on Indian numbers." Soon European scholars recognized the value of al-Khwārizmī's treatise and there appeared more treatises in Latin elaborating al-Khwārizmī's treatise. What needs to be emphasized in this context is that the symbol for zero known to al-Khwārizmī and was transmitted through him to Europe was a small circle.[28] Indeed zero is called in Arabic, besides *al-ṣifr,* also by the expression *dā'ira ṣaghīra* (small circle). And it is this small circle (*circulus*) which appears in early Latin manuscripts.[29] While zero retained its circular form in its transmission from the Eastern Islamic world to the Western Islamic world, the digits from 1 to 9 gradually underwent several changes, with the consequence that the western forms substantially differed from the eastern ones. These western forms came to be known as *Ghubār* numerals. The eastern forms were transmitted to Italy and the Eastern Mediterranean basin where they were used by Latin authors in the twelfth century. By the early thirteenth century these eastern forms were replaced in Europe by the western forms through the Latin translations made in the Moorish Spain.[30] These then are the ancestors of what we call today "Arabic" numerals (or what the Constitution of India, more accurately and wisely terms, in article 343.1, as "the international form of Indian numerals"[31]).

As mentioned earlier, the zero retained its form in Europe in the shift from eastern to the western numerals. But in the Arabic script, apparently at a later point, this circle was compressed into a dot, in order to distinguish the symbol for zero from that of five —a process that is the reverse of what happened in India.

But the westward transmission of Indian mathematical ideas was not limited to the number system alone. It has been stated that the main areas which influenced the future course of development of mathematics are (1) the spread of Indian numerals and their associated algorithms, first to the Arabs and later to Europe; (2) the spread of Indian trigonometry, especially the use of the sine function, and (3) the solutions of equations in general, and of indeterminate equations in particular. In this context, I may invite attention to a special case of transmission of a rather simple procedure of calculation, namely *Trairāśika* or the Rule of Three.

3. Development and Transmission of the Rule of Three

Though normally employed in solving commercial problems, such as computing the price of *a* mangoes, if the price of *b* mangoes is known as *c* Rupees, *Trairāśika* or the Rule of Three plays a far more important role in Indian mathematics and astronomy.[32] In arithmetic it is often used as a means of verification in solving other problems. More importantly, it is employed in astronomical computations, for example, in the computation of the mean position of a planet from the number of its revolutions in a *kalpa* of 4,320,000,000 years. Many of the problems of spherical trigonometry are solved by applying the Rule of Three to similar triangles (*akṣakṣetra*). Likewise, the Rule of Three forms the basis for computing trigonometric ratios. Therefore Nīlakaṇṭha Somasutvan declares in his commentary on the *Āryabhaṭīya* that the entire mathematical astronomy (*graha-gaṇita*) is pervaded by two fundamental laws: by the law of relation between the base, perpendicular and hypotenuse in a right-angled triangle—which goes today under

the name of Pythagoras theorem— and by the Rule of Three (*bhujakoṭikarṇanyāyena trairāśikanyāyena cobhābhyāṃ sakalaṃ grahagaṇitaṃ vyāptam*).

Therefore it would be interesting to trace its development and spread. In India the Rule was first mentioned by Āryabhaṭa in his *Āryabhaṭīya* in 499 AD.[33] Here Āryabhaṭa not only gives the name *Trairāśika* (that which consists of three numerical quantities or terms) for the Rule of Three, he also mentions the technical terms for the four numerical quantities involved (*pramāṇa, phala-rāśi, icchā-rāśi, icchā-phala*) and gives the formula for solving the problem. Subsequent writers, notably Brahmagupta in his *Brāhmasphuṭasiddhānta* (628 AD) and Bhāskara I in his commentary (629 AD) on the *Āryabhaṭīya* elaborate upon this brief statement by Āryabhaṭa, but employ the same terminology, albeit with slight modifications. It is on the basis of the writings of these mathematicians that histories of mathematics generally trace the origin of the Rule of Three to India.

The brief manner in which Āryabhaṭa presents the rule in his work implies that he is referring to an already well known rule which he is restating in order to employ it in astronomical computations. Therefore, it is tempting to look for the antecedents for Āryabhaṭa's rule.

Kuppanna Sastry sees the first mention of the Rule of Three in a verse of the *Vedāṅgajyotiṣa* (Rk-recension 24; Yajus-recension 42).[34] Obviously, we have here a rudimentary form of the Rule of Three and it is also obvious that the rule was needed for the computations envisaged in the text. Although Indians developed special terminology for the Rule of Three in later times, the general terms *jñāna<ta>rāśi* (the quantity that is known or given), *jñeya-rāśi* (the quantity that is to be known) used here are also frequently employed in later times. Indeed it is conceivable that the term *jñāna* gave rise to the later term *pramāṇa*. However, the date of this text, available in two recensions, is uncertain. Kuppanna Sastry himself would like to place the composition of the text in the period between 1370 – 1150 BC; others assign it to 500 BC. In either case, Āryabhaṭa's rule appears to have a long pre-history in India.

However, Joseph Needham observes that the "Rule of Three, though generally attributed to India, is found in the Han *Chiu Chang*, earlier than in any Sanskrit text. Noteworthy is the fact that the technical term for the numerator is the same in both languages --- *shih* and *phala*, both meaning 'fruit'. So also for the denominator, *fa* and *pramāṇa*, both representing standard unit measures of length." Needham goes on to add that "Even the third known term in the relationship can be identified in the two languages. For *icchā*, 'wish, or requisition' reflects *so chhiu lü*, i.e. ratio, the number sought for."[35]

Writing in the *Gaṇita-Bhāratī,* N. L. Maiti, draws attention to the passage of the *Vedāṅga Jyotiṣa*, in order to counter Needham's claim of Chinese priority.[36] Maiti also disputes Needham's linguistic equation *fa = pramāṇa; shih = phala; so chhiu = icchā.* Finally, he tries to clinch the issue by citing the view of a Chinese scholar Lam Lay Lang to the effect that "the Rule of Three … originated among the Hindus …"[37]

I am not competent to judge in favour or against Needham's linguistic equation, but there is no denying the fact that the Rule of Three had an important place in Chinese mathematics as well. Even if the verse from the *Vedāṅgajyotiṣa* alludes in a rudimentary form to the Rule of Three and thus testifies to the existence of the rule in the centuries before the Christian era, there is nothing to prevent the knowledge of the Chinese *Chiu Chang* to travel to India in the early centuries of the Christian era and to give impetus to the development of the Rule in India. If India received impetus from China in this process of development and then transmitted an elaborate system to the Middle East and Europe, then this would testify to both the receptivity and creativity in mathematical thought in India. It would, however, be nice if the transmission could be mapped in detail.

By the time of Brahmagupta in the early seventh century, the Rule of Three and its variations reached their full development. In the next century, various elements of Indian mathematics and astronomy were disseminated to the Islamic world. The Rule of Three seems to be one of the elements thus transmitted. From the ninth century onwards, Arab mathematicians began to discuss the Rule of Three and other variants.

Thus Al-Khwārizmī discusses the Rule of Three in his book on Algebra. This treatise contains a small chapter on commercial problems including the simple Rule of Three according to the Indian model.[38] Al-Bīrūnī (973-1048) composed an exclusive tract on the Rule of Three entitled *Fī Rāshikat al-Hind,* where he discusses direct and inverse Rule of Three as well as the rules for five, seven and more terms up to seventeen.[39] Together with Indian numerals and commercial problems, the Rule of Three was transmitted to Europe where it was hailed as the Golden Rule.[40]

This case too exemplifies that while it is possible to trace the path of transmission between India and the West, the historiography of mathematics has not yet reached a stage where it could clearly define the interaction between Chinese and Indian mathematics and astronomy.

4.0 Transmission of Perpetuum mobile

I should like to discuss another case where linguistic limitations and national or personal biases obstruct the correct apprehension of the transmission of ideas, this time of technical nature.[41] In the context of non-European contribution to the development of technological ideas in Eurpe, Lynn White brought out significant studies on medieval technology. Notable in this connection is his seminal essay *Tibet, India, and Malaya as Sources of Western Medieval Technology.*[42] One of the concepts whose origin he attributes to India is the perpetual motion machine. Lynn White avers that the perpetual motion machines designed by Bhāskara in the twelfth century were instantly accepted by the Islamic world and then transmitted to Europe, where people like Villard de Honnecourt, in their quest for energy, received this notion with great interest and tried to apply it to the benefit of mankind. Thus, concludes White, were laid the foundations for the power technology of the modern world.

Lynn White's thesis was generally accepted by historians of technology, but his attempt to trace the origin of the perpetual motion machine to twelfth-century India was contested by two sides, the former contending that such machines were known to the Arabs before Bhāskara's time and the latter claiming that both the Indian and Arabic accounts owe their inspiration to China.

Ahmad Y. Al-Hassan and Donald R. Hill argue in their excellent book *Islamic Technology: An Illustrated History* thus: "In India about A. D. 1150 Bhāskara described a perpetual motion wheel which resembles one of the six such wheels in the Arabic manuscripts, but the original Arabic text is of an earlier date. The Arabic technical descriptions, the illustrations, and the whole complex of the sixteen machines are quite elaborate and, as we have seen, constitute a single approach. The occurrence, therefore, of one or two perpetual-motion wheels in the Indian text does not imply a case of transmission from one culture to another, though there was an important transmission to the West."[43] About the original Arabic text, the authors maintain: "It must have been copied from an original treatise which is at present unknown to us. We can tell, however, that this original was written between the third and sixth centuries AH (ninth to twelfth centuries AD)."[44] But it is possible to show that the Indian concept of the *perpetuum mobile* is much older than Bhāskara and also older than the alleged antiquity of the "unknown and undated" original Arabic text.

The second party of opposition to Lynn White's view is represented by Joseph Needham, who asserts that "Indeed one begins to entertain the belief that the stimulus for the flood of ideas on the perpetual motion devices may have been derived from Indian monks or Arabic merchants standing before a clock tower such as that of Su Sung and marvelling at its regular action."[45] Lynn White dismissed this suggestion as lacking in any evidence.[46]

The astonishing thing about this debate—like many other debates concerning India's past—is that it is conducted on the basis of just two Sanskrit texts which happen to be available in English translation, ignoring all other texts. Lynn White traces the idea of the perpetuum mobile to twelfth-century India on the basis of Lancelot Wilkinson's translation of the *Siddhāntaśiromaṇi*,[47] while Needham's comments emanate from his perusal of Ebenezer Burgess's rendering of the *Sūryasiddhānta*.[48] The passage cited by Needham does not even discuss the perpetuum mobile. No doubt, Lynn White's conclusions are highly perceptive even with the limited sources available to him, but in history of

technology there are no shortcuts. One has to study all the relevant original texts, and the material remains if there are any, and then interpret the data in the correct space-time framework. In the present case, a study of the original texts not only upholds Lynn White's view, but strengthens it further. For a perusal of the Sanskrit sources shows that the idea of perpetual motion is much older in India than Bhāskara's time and that the philosophical notion was translated into a design for a mechanical instrument by Brahmagupta in the early seventh century. Brahmagupta's mercury wheel is not only earlier than Su Sung's clock tower (1090 AD) but also works on a totally different principle. On the other hand, there is much in common between Brahmagupta's wheel and those found in Arabic manuscripts, for both are supposed to be driven by mercury power. As mentioned earlier, Brahmagupta's writings on astronomy and mathematics were transmitted to the Islamic world in the eighth century and these may have included the design for the perpetual motion machine as well.

"In today's world of narrow loyalties, one is accustomed to ask to whom the credit should go: is it due to Brahmagupta for the origin of the idea, or to the Islamic world for its elaboration and spread, or to the Occident for its practical application? Lynn White, quite rightly, sees these three kinds of endeavour as complementary to one another."[49]

5.0 Transmissions within India

While the study of transmissions from and to India has its own importance, I think it is also necessary to study the transmission of ideas within India, from one region to another, from Sanskrit to regional languages and vice versa.

On the one hand, we have the cases of swift transmission of ideas from one end of the country to the other. In 1185 AD, in his commentary on the *Sūryasiddhānta*, Caṇḍīśvara of Mithila cites another commentary on the same text written by Mallikārjuna Sūri in the distant Telugu region just 7 years previously in 1178 AD.[50] On the other hand, certain texts like the *Candra-vākyas* of Vararuci do not seem to have been transmitted beyond Tamilnadu.[51] Nearly all the surviving manuscripts of the *Āryabhaṭīya* are only in Malayalam script.

Again, certain texts, indeed very valuable ones, are irretrievably lost, although there had been no significant break in the study of mathematics and astronomy. It is greatly intriguing why Āryabhaṭa's book on the midnight reckoning system is wholly lost; why Bhāskara's excellent commentary on the *Āryabhaṭīya* is just partially preserved; and why the equally excellent commentary on Śrīdhara's *Pāṭīgaṇita* is not completely available. Indeed, Mādhava's contributions to the power series are known only as citations from later writers; what happened to Mādhava's original works ?

Even such a useful tool like the *Kaṭapayādi* notation, which greatly facilitated the development and spread of astronomical and mathematical tables in Kerala, did not spread or spread very slowly to other parts of India.[52] That it was employed in Kerala very widely, not only in works on astronomy and mathematics, but also in non-scientific works, not only in Sanskrit writings but in Malayalam as well, is now quite well established.[53] From Kerala, the *Kaṭapayādi* system spread to the neighbouring Tamilnadu, where its use is fairly well attested.[54] However, no literary works from the regions of Karnataka or Andhra have been identified so far which employ this system of notation.

Towards the middle of the tenth century Āryabhaṭa II used a variant[55] of the *Kaṭapayādi* system in the first thirteen chapters of his *Mahāsiddhānta*. Here the digits are read from the left to the right and every member of a conjunct consonant has a numerical value.[56] But we do not know where this Āryabhaṭa hailed from. Two centuries later, in Maharashtra, Bhāskara II employed this notation a few times in his commentary on the *Śisyadhīvṛddhida*,[57] but not in any other work of his. In North India proper, I know of only two instances. These occur in the works of Rāmacandra Vājapeyin[58] and his brother Harṣa,[59] who flourished in the first half of the fifteenth century in Naimiṣāraṇya, close to modern Sitapur in Uttar Pradesh. These are the only instances from literary documents. The *Bhūtasaṃkhyā*[60] system of word numerals is widely employed in inscriptions,[61] but the use of the *Kaṭapayādi* system is limited to very few records, that too belonging to the 14-16th centuries.[62]

While cataloguing pre-modern Indian astronomical instruments, I came across two Sanskrit instruments that carry the *Kaṭapayādi* notation. In Islamic astrolabes and celestial globes, it is customary to inscribe the numbers on the scales in an alphabetical notation called *Abjad*. Clearly in imitation of this, an undated Sanskrit astrolabe, now preserved in the Sanskrit University at Varanasi, displays *Kaṭapayādi* notation in some of the scales.[63] In the middle of the nineteenth century, a certain Bhālumal of Lahore, who made astrolabes and celestial globes in Arabic as well as in Sanskrit, produced a Sanskrit celestial globe on which the scale on the horizontal ring was labelled in the *Kaṭapayādi* notation. This globe is now in a private collection in Milan.

Likewise, transmissions from Sanskrit to regional languages and vice versa remain an almost unexplored area. Thanks to the bibliographical work of Professor K. V. Sarma, we know about mathematical and astronomical works in Malayalam language, but very little is known about such texts in other Indian languages. Without exploring the literature in regional languages, a full picture will not emerge of how mathematical ideas developed and systematized in Sanskrit manuals and how they were disseminated and popularized in the regional languages.[64]

DISCUSSION

This paper examines some cases of possible early transmissions of Indian mathematical ideas (the decimal place-value number system with zero, the rule of three and the notion of perpetual motion machines) to other cultures – in particular Chinese, Arabic and Western civilizations. It also tries to address some epistemological and methodological issues concerned with documenting and establishing such occurrences. Moreover, given the geographical extent and linguistic diversity of India, it points to the importance of studying how mathematical ideas were disseminated historically within India itself. The paper's general conclusion is that although ample documentation exists for Indian transmissions to the Arab world, and thence to the West, there is much less documentation or study of other transmissions. It recommends that more attention be paid to them in the future.

The paper's key contribution appears to be the controversial claim that the decimal place-value system with zero developed in India before the beginning of the Christian era rather than in the fifth century as is often supposed. However, the argument made for this realignment of mathematical history does not stand close scrutiny. To see why such a conclusion is warranted let us examine case made by the paper more carefully.

The paper begins by identifying the distinct elements constituting the Indian number system - counting with base ten, the notions of zero and place value, and the symbols used to represent the nine digits and zero. All of these elements, the paper argues, can be traced to India before the Christian era. Firstly, decuple terms occur in Hindu and Jain religious canonical texts in that period proving that Indian mathematics adopted a decimal system.

Secondly, the term *śūnya* occurs in Pingala's *Chandahsutra* in the context of teaching a method for computing the number of permutations of a verse containing a certain number of syllables, where Pingala uses the symbol for *sunya* to denote the *absence* of the mathematical operation of halving. The paper goes on to ask "What kind of mathematical concept lay behind this symbol for *sunya*? From Pingala's use, it may appear that *śūnya* meant here the absence of an operation, akin to the grammarian's *lopa*. But is that all, or does *śūnya* here imply place value as well?"

However, instead of answering this question the paper appeals to Needham's view that the zero symbol could not have come into being without knowledge of place value. The inference is that Pingala's employment of the zero symbol presupposes knowledge of place value. Consequently, the decimal place value system with the concept of zero must have been known to Pingala.

However, it could be argued that the paper presupposes what it sets out to prove – namely that Pingala had the concept of zero. Even if Needham rightly assumes that the use of the symbol *sunya* for zero presupposes knowledge of place-value, Pingala's use of the symbol *sunya* for an absence of an operation does not need to presuppose the concept of place-value. Hence

Pingala's use of the symbol *sunya* to denote 'absence of an operation' cannot be used to infer that he had knowledge of the concept of zero.

ENDNOTES

---

1 Thus Muḥammad ibn Mūsā al-Khwārizmī (ca. 780-850) calls his book *Kitāb al-jam'wal tafrīq bi ḥisāb al-Hind,* "Book on Addition and Subtraction after the Method of the Indians" and Abul l-Ḥasan al-Uqlīdīsī his book, written in 952, *Kitāb al-fuṣūl fi-l-ḥisāb al-Hindī,* "The Book of Chapters on Indian Arithmetic".

2 Fuat Sezgin, *Wissenschaft und Technik im Islam,* Frankfurt, 2003, Vol. 1, p. 142: "In der bisherigen Historiographie der Wissenschaften wurde leider zu wenig beachtet, dass das Zitieren von Quellen eine der charakeristischen Eigenschaften des arabisch-islamischen Schrifttums ist, auch wenn dies nicht bedeutet, dass es dort keine Plagiate gegeben hätte oder sich jeder Schriftsteller an die allgemeine Regel gehalten hätte."

3 *Bṛhatsaṃhitā,* (ed. Avadh Vihari Tripathi, Varanasi, 1968), 2.75:
mlecchā hi yavanās teṣu samyak śāstram idaṃ sthitam /
ṛṣivat te 'pe pūjyante kiṃ punar daivavid dvijaḥ //

4 *Yantrarāja,* (ed. Kṛṣṇaśaṅkara Keśavarāma Raikva, Bombay, 1936), 1.3:
kḷuptās tathā bahuvidhā yavanaiḥ svavāṇyā yantrāgamā nijanijapratibhāviśeṣāt /
tān vāridhīn iva viloḍya mayā sudhāvat tatsārabhūtam akhilaṃ praṇipadyate 'tra //

5 David Pingree, *Jyotiḥśāstra: Astral and Mathematical Literature,* Wiesbaden 1981, pp. 95-100.

6 While the transformation of Sanskrit *jīvā* into Arabic *jayb* is understandable, it is difficult to account for the change of *śūnya* to Arabic *ṣifr*.

7 Needless to say that mere phonetic similarity of certain terms does not establish the transmission of ideas unless it is supported by other evidence.

8 A. Weber, *Die Vedischen Nachrichten von den Naxatra (Monstationen),* part 2, Berlin, 1860-1862, pp. 362-63. The *Report of the Calendar Reform Committee,* New Delhi, 1955, pp. 225-26, and Kane, *History of Dharmaśāstra.* Vol. I, part 1. Poona, 1958, pp. 541-43, contest this view, with arguments already anticipated by Weber. Kane asserts that any astronomer living in Gandhāra could have arrived at the longest day of 18 *muhūrtas* after a few

years of observations without having to borrow the idea from Babylonia. While that is possible, there is no record of maximum and minimum values from other latitudes, say from that of Ujjain. It is striking that all these texts (and several *purāṇas* cited by Kane, p. 41) should consistently speak only of the longest day of 18 *muhūrtas*, without allowing for changes according to the local latitude, as though they were repeating a hoary tradition rather than their own observations.

9 Therefore, I particularly appreciate efforts of the Āryabhaṭa group in looking for evidence that is "legally binding" for the transmission of Kerala mathematics to Europe. Although the possibilities do exist, the path of transmission can be traced and possible mediators can be identified, still it is necessary to show that the transmission did indeed take place.

10 K. V. Sarma & B. V. Subbarayappa, *Indian Astronomy : A Source-Book,* Bombay, 1985, pp. 46-47; Takao Hayashi (ed. & tr), *The Bakhshālī Manuscript: An ancient Indian mathematical Treatise,* Groningen, 1995, p. 66.

11 Bibhutibhusan Datta & Avadhesh Narayan Singh, *History of Hindu Mathematics: A Source Book,* Bombay, 1935, 1939; reprint, 1962, part I, pp. 75-77; see also S. R. Sarma, "Śūnya, Mathematical Aspect" in: Bettina Bäumer (ed), *Kalātattvakośa: A Lexicon of Fundamental Concepts of the Indian Arts,* vol. 2, New Delhi, 1992, pp. 400-411; idem, "Śūnya in Piṅgala's Chandaḥsūtra" in: A. K. Bag & S. R. Sarma (ed), *The Concept of Śūnya,* New Delhi, 2003, pp. 126-136.

12 Joseph Needham & Wang Ling, *Science and Civilisation in China,* Vol. III: Mathematics and Sciences of the Heavens and the Earth, Cambridge, 1959, p. 10 n.

13 Johannes Bronkhorst, "A Note on Zero and the Numerical Place-Value System in Ancient India," *Asiatische Studien /Études Asiatiques,* 48.4 (1994) 1039-1042.

14 S. R. Sarma, "Śūnya in Piṅgala's Chandaḥsūtra," op. cit., pp. 126-136, esp. 132-133.

15 G. Coedès, "A propos de l'orgine des chiffres arabes." *Bulletin of the School of Oriental and African Studies,* 6 (1930-32) 323-328.

16 First published in *Epigraphia Indica* II, p. 19. For discussion, see Datta & Singh, op. cit., pp. 40-51. See also A. M. Shastri, "Mankhani Charter of Taralasvāmin and the Antiquity of Decimal Notation," *Annals of the Bhandarkar Oriental Research Institute,* 79 (1998) 160-177; Irfan Habib, "Joseph Needham and Indian Technology," *Indian Journal of History of Science,* 35 (2000) 245-

274, esp. 272-274; Georges Ifrah, *The Universal History of Numbers, From Prehistory to the Invention of the Computer,* translated from the French by David Bellos et al, New York, 2000, pp. 356-439, esp. p. 402 and figure 24.76.

17 Joseph Needham & Wang Ling, *Science and Civilisation in China,* Vol. III, Cambridge, 1959, pp. 10-11.

18 Takao Hayashi (ed. & tr), *The Bakhshālī Manuscript: An ancient Indian mathematical Treatise,* Groningen, 1995, esp. p. 89.

19 *The Patiganita of Sridharacarya, with an ancient Sanskrit Commentary,* edited by Kripa Shankar Shukla, Lucknow, 1959, Introduction, p. xxix.

20 A parallel situation obtains in the case of the expressions *nālikā* and *ghatikā*. The first expression meant originally an outflow water clock of cylindrical shape and time unit of 24 minutes measured by this clock. Around the fourth century AD, this type of water clock was replaced by the sinking bowl type of water clock which was called *ghaṭikā*. The time unit of 24 minutes measured by this variety of water clock was called *ghaṭikā*. But *nālikā* the older designation for this time unit also continued to be used. Often both are used as synonyms in the same text. In the South, however, *nālikā* survives in Tamil and Malayalam (with slight phonetic modification) while *ghaṭikā* (with slight phonetic change) survives in Kannada and Telugu.

21 S. N. Sen, "Transmission of Scientific Ideas between India and Foreign Countries in Ancient and Medieval Times," *Bulletin of the National Institute of Sciences of India,* No. 21 (1962) 8-30; idem, "Influence of Indian Science on Other Culture Areas," *Indian Journal of History of Science,* 5 (1970) 332-346; idem, "India and the Ancient World: Transmission of Scientific Ideas" in *The Cultural Heritage of India,* volume VI: Science and Technology, edited by Priyada Ranjan Roy & S. N. Sen, Calcutta, 1986 (reprint 1991), pp. 220-247; R. C. Gupta, "Indian Mathematics Abroad upto the Tenth Century A.D.," *Gaṇita-Bhāratī,* 4 (1982) 10-16; idem, "Sino-Indian Interaction and the Great Chinese Buddhist Astronomer-Mathematician I-Hsing (A.D. 683-727)," ibid, 11 (1989) 38-49. See also Wilhem Rau, *Indiens Beitrag zur Kultur der Menschheit,* Sitzungsberichte der Wissenschaftlichen Gesellschaft an der Johann Wolfgang Goethe-Universität Frankfurt am Main, Band III, Nr. 2, Wiesbaden, 1975.

22 Duan Yaoyong investigated this question in his master's thesis entitled "The Discussion that Indian Trigonometry affected the Chinese Calendar Calculation in Tang Dynasty (A.D. 618-907)",

Institute for the History of Science, Inner Mongolia Normal University, 1996, and came to the following conclusion: "I undertook a systematic study and organization of this topic for the first time (some made fragmentary study about it before); got that trigonometry knowledge from English translation of Indian Astronomy Corpus and tried to make it systematized (nobody did it before). I found an equivalent way that is equal to the method of trigonometry in China; made study of Chinese and Indian calendric calculation system, and tried to find the internal relations of them. I answer Dr. J. Needham's question [viz. 'How much trigonometry did the Tang astronomers know and where did they learn it?' with the reply that] 'Chinese calculation has nothing to do with Indian trigonometry. Chinese algorithm specialist solved the problem with 'the gougu theorem' which is equivalent way of trigonometry used by Indians,' trying to set out reasons why Chinese never accepted trigonometry." Cf. Thesis Abstract, *Gaṇita-Bhāratī: Bulletin of the Indian Society for History of Mathematics,* 20 (1998) 110-111.

23 *Alberuni's India,* tr. Edward C. Sachau, reprint: Delhi, 1964, vol. 2, p. 15.

24 S. N. Sen, "Transmission of Scientific Ideas between India and Foreign Countries in Ancient and Medieval Times," *Bulletin of the National Institute of Sciences of India,* No. 21 (1962) 8-30, esp. 24-25.

25 David Pingree, *Census of Exact Sciences in Sanskrit,* A-2, Philadelphia, 1971, s. v. Kanaka, p. 19.

26 In the fourteenth century Arab navigators report of their meeting the "Kanaka" on the Malabar coast, who must likewise be "Gaṇakas", i.e., astronomers or more specifically navigators. Personal communication from Dr Farid Benfeghoul, Institut für Geschichte der Arabisch-Islamischen Wissenschaften, Frankfurt.

27 See, inter alia, George Gheverghese Joseph, *The Crest of the Peacock,* pp. 311-12; see also Edgar Reich, "Ein Brief des Severus Sebokt" in: Menso Folkerts & Richard Lorch (ed), *Sic ad Astra: Studien zur Geschichte der Mathematik und Naturwissenschaften: Festschrift für den Arabisten Paul Kunitzsch zum 70. Geburtstag,* Wiesbaden, 2000, pp. 478-489.

28 "Algorizmi said: when I saw that Indians composed out of .IX. letters any number due to the position established by them, I desired to discover, God willing, what becomes of those letters to make it easier for the student ... Thus, they created .. IX. letters, figures of them are as follows ... The beginning of the order is on the right side of the writer, and this will be the first of them

consisting of unities. If instead of unity they wrote .X. and it stood in the second digit, and their figure was that of unity, they needed a figure of tens, similar to the figure of unity so that it became known from it that this was .X. Thus they put before it one digit and wrote in it a small circle "o", so that it would indicate that the place of unity is vacant." Cited in: B. A. Rosenfeld, "Al-Khwārizmī and Indian Science" in: W. H. Abdi et al (ed), *Interaction between Indian and Central Asian Science and Technology in Medieval Times,* Indian National Science Academy, New Delhi, 1990, vol. I, pp. 132-139, esp. 132.

29 See Charles Burnett, "Indian Numerals in the Mediterranean Basin in the Twelfth Century, with Special Reference to the 'Eastern Forms'" in: Yvonne Dold-Samplonius et al (ed), *From China to Paris: 2000 Years Transmission of Mathematical Ideas,* Stuttgart, 2002, pp. 237-288, esp. 239, 265-267.

30 Ibid.

31 This is no doubt an elegant solution for the nomenclature of the numerals of the form 1, 2, 3 etc. But how does one designate unambiguously the numerals associated with the Arabic-Persian script, such as ١٢٣ ? Again, when the Indian numerals were transmitted to Europe, they were first known as the "Indian" numerals. One wonders when Europe started calling them "Arabic" numerals and why.

32 S. R. Sarma, "Rule of Three and its Variations in India" in: Yvonne Dold-Samplonius et al (ed), *From China to Paris : 2000 Years Transmission of Mathematical Ideas,* Stuttgart, 2002, pp. 133-156.

33 *Āryabhaṭīya, Gaṇitapāda* 26.

34 *Vedāṅga Jyotiṣa of Lagadha in its Ṛk and Yajus Recensions*, with the Translation and Notes of T. S. Kuppanna Sastry, ed. by K. V. Sarma, New Delhi, 1985, pp. 40-41.

35 Joseph Needham & Wang Ling, *Science and Civilisation in China,* Vol. III, Cambridge, 1959, p. 146.

36 N. L. Maiti, "The Antiquity of Trairāśśika in India," *Gaṇita-Bhāratī* 18 (1996) 1-8.

37 Lam Lay Yong, *A Critical Study of the Yang Suan Fa*, Singapore, 1977, p. 329: "The Rule of Three which originated among the Hindus is a device used by oriental merchants to secure results to certain numerical problems."

38 Adolf P. Juschkewitsch, *Geschichte der Mathematik im Mittelalter,* Leipzig, 1964, p. 204.

[39] Ibid, p. 214. Al-Bīrūnī, *Fī Rāshikāt al-Hind*, in: *Rasá'ilu'l-Bírúní by Abú Rayhan Muh. b. Ahmad al- Bírúní* ... The Osmania Oriental Publications Bureau, Hyderabad-Dn., 1948.

[40] See, inter alia, D. E. Smith, *History of Mathematics*, II, p. 486 ff.

[41] S. R. Sarma, "Perpetual Motion Machines and their Design in Ancient India," *PHYSIS: Rivista Internationale di Storia della Scienza*, Rome, 29.3 (1992) 665-676.

[42] L. White Jr., "Tibet, India, and Malaya as Sources of Western Medieval Technology," *American Historical Review*, 65 (1960), pp. 515-526; reprinted in: idem, *Medieval Religion and Technology. Collected Essays*, Berkeley, 1978, pp. 43-57. See also, idem, *Medieval Technology and Social Change*, London, 1964, pp. 19-131.

[43] A. Y. Al-Hassan, D. R. Hill, *Islamic Technology. An Illustrated History*, Cambridge-Paris, 1985, p.71.

[44] Ibid., p. 70.

[45] J. Needham, *Science and Civilization in China*, vol. IV, part 2, Cambridge, 1965, p. 540.

[46] L. White Jr., *Medieval Religion*, op. cit., p. 53, n. 60 (= idem, *Medieval Technology*, op. cit., p. 130, n, 3).

[47] For the text, see Bhāskara, *Siddhāntaśiromaṇi*, ed., by Bapu Deva Sastri, rev. by Ganpati Deva Sastri, Benares, 1920, Golādhyāya, Yantrādhyāya, vv. 50-53.

[48] For the text, see *The Sūryasiddhnta*, with the Exposition of Ranganātha, the Gūḍhārtha-prakāśkā, ed. by F. Hall, reprint, Amsterdam, 1974, 13.16-18.

[49] S. R. Sarma, "Perpetual Motion Machines and their Design in Ancient India," op. cit.

[50] David Pingree, *Census of Exact Sciences in Sanskrit*, A 3, Philadelphia, 1971, pp. 40-41, s. v. Caṇḍīśvara; A 4, Philadephia, 1981, p. 368, s. v. Mallikārjuna Sūri.

[51] In 1825, at Pondichery, one Swaminathan Seshayya showed John Warren how to compute the lunar eclipse of 31 May-1 June 1825 with the help of these *Vākyams*; cf. John Warren, *Kālasamkalita*, pp. 334-340: Fragment IV: "Computation of an Eclipse of the Moon by means of certain memorial and artificial words, and of shells in lieu of figures ... By Sami Naden Sashia, a Kalendar maker residing in Pondichery." In their study of this material collected by John Warren, Neugebauer and Van der Warden coined the phrase "Tamil Astronomy" which is actually "Kerala Astronomy".

[52] S. R. Sarma, "On the Spread of the *Kaṭapayādi* System outside Kerala," forthcoming.

[53] K. V. Sarma, "Word and Alphabetic Numerical Systems in India," in: A. K. Bag & S. R. Sarma (ed), *The Concept of Śūnya*, New Delhi, 2003, pp. 37-71, esp. 43-44.

[54] K. V. Sarma, "Word and Alphabetic Numerical Systems in India," op. cit., p. 44.

[55] Datta & Singh, *History of Hindu Mathematics*, Part I, pp. 69-72, assert that there were four variants of the *Kaṭapayādi* system, and this is repeated by everybody. In fact there were only two variants, viz. the standard system as practised in Kerala and the one employed by Āryabhaṭa II.

[56] S. R. Sarma, *The Pūrvagaṇita of Āryabhaṭa's (II) Mahāsiddhānta*, Marburg 1966; see esp. Part I, pp. xx-xxii.

[57] *Śiṣyadhīvṛddhidam of Lallācārya, with the Commentary Vivaraṇa by Śrīmad Bhāskarācārya*, ed. Chandra Bhanu Pandey, Sampurnanand Sanskrit University, Varanasi 1981.

[58] On Rāmacandra Vājapeyin, see S. R. Sarma, "Astronomical Instruments in Mughal Miniatures," *Studien zur Indologie und Iranistik*, 1992, 16-17, pp. 235-276; David Pingree, *Census of Exact Sciences in Sankrit*, series A, Volume 5, Philadelphia, 1994, pp. 467-478; S. R. Sarma, "On the Life and Works of Rāmacandra Vājapeyin," forthcoming.

[59] On Harṣa, see G. V. Devasthali, "Harṣa, the author of the Anka-yantra-cintāmaṇi and Relatives," *B. C. Law Volume*, part 1, Calcutta 1943, pp. 496-503.

[60] Incidentally, this expression *Bhūtasaṃkhyā* appears to be a recent coinage; it is not attested in any Sanskrit text.

[61] D. C. Sircar, *Indian Epigraphy*, Delhi, 1965, pp. 228-233.

[62] G. H. Ojha, *Bhāratīya Prācīna Lipimālā*, p. 123; see also Datta & Singh, op. cit., pp. 70-71.

[63] S. R. Sarma, "Kaṭapayādi Notation on a Sanskrit Astrolabe," *Indian Journal of History of Science* 34 (1999) 273-287.

[64] S. R. Sarma, "Mathematical Literature in the Regional languages of India," forthcoming.